Math 3A: Exam 2

Directions: Show your work! Any problem with a final answer but no work shown will automatically receive a score of zero.

1. True or False.   (5 points total)

T   F   If $f'(c) = 0$ and $h(x) = f(x)g(x)$, then $h'(c) = 0$.

T   F   Newton's method fails when the initial guess x_o corresponds to a horizontal tangent line for the graph of f at x_o

T   F   If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.

T   F   All absolute extrema are also local extrema.

T   F $\frac{d}{dx}[\cos(y^2)] = -2y \cdot \sin(y^2)$

2. Prove the following: (4 pts)

a) $\frac{d}{dx}[\tan x] = \sec^2 x$

b) $\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$

(you can use derivative here)

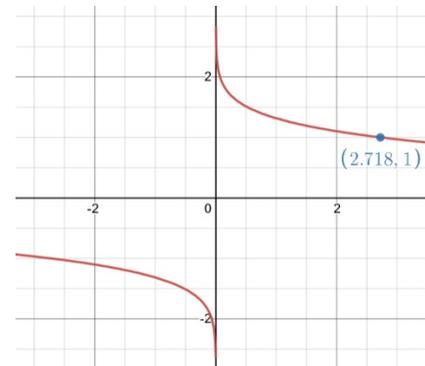3. Find y' for each of the following functions: (9 points total)

a) $y = \cot^6\left(\sqrt[3]{\arcsin(9x)}\right)$

b) $y = \dfrac{8^{\pi x} \cdot \ln(x)}{\text{arccot}(4x)}$

c) $y = (x - 2)^{x+1}$

4. Consider the implicit function $y^2 + \ln(xy) = 2$. (5 points total)

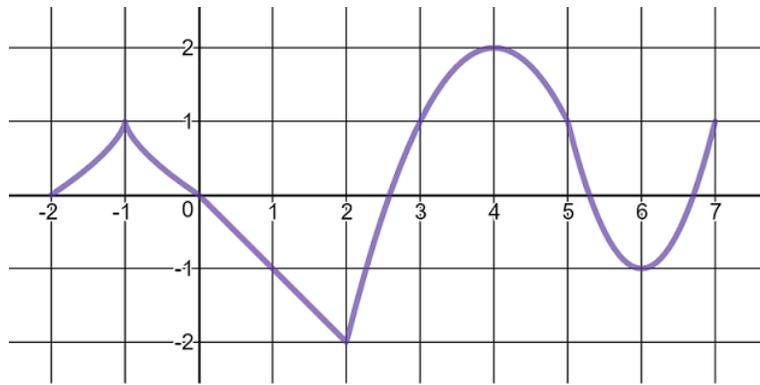   a) Find y'. (3 pts)

(2.718, 1)

   b) Find the tangent line to the graph at the point $(e, 1)$. (2 pts)

5. Find $(f^{-1})'(2)$ if $f(x) = \frac{x+3}{x+1}$. (4 points total)

   Note: You must use the method in section 3.6. Do not find the inverse function and differentiate it.

6. Use the graph below to answer the following questions: (4 pts total)

a) List all x values where $f'(x)$ does not exist. (1 pt)

b) List all local minimum and local maximum as a point (x,y). (2 pt)

Local Minimum: Local Maximum:

c) State the absolute minimum and maximum of the function over the interval $[0, 3]$. (1 pt)

Absolute Minimum: Absolute Maximum:

7. Find the absolute maximum and minimum of the function on the indicated interval. (5 points)

$$f(x) = x\sqrt{1 - x^2} \qquad\qquad [-1,1]$$

8. The ordering and transportation cost C for components used in a manufacturing process is
   approximated by the function   (4 pts)

$$C(x) = 10 \left(\frac{1}{x} + \frac{x}{x+3} \right)$$

Where C is measured in thousands of dollars and x is the order size in hundreds.

a) Verify that $C(x)$ satisfies Rolle's theorem on the interval $[3, 6]$. (2 pts)

b) Apply Rolle's theorem to show that the rate of change of the cost must be zero for some order size
   in the interval $(3, 6)$. Find that order size. (2 pts)

9. Sketch the secant line through the endpoints of the interval $[a, b]$ and sketch any tangent lines to the
   graph for each value c guaranteed by the Mean Value Theorem. (3 pts total)
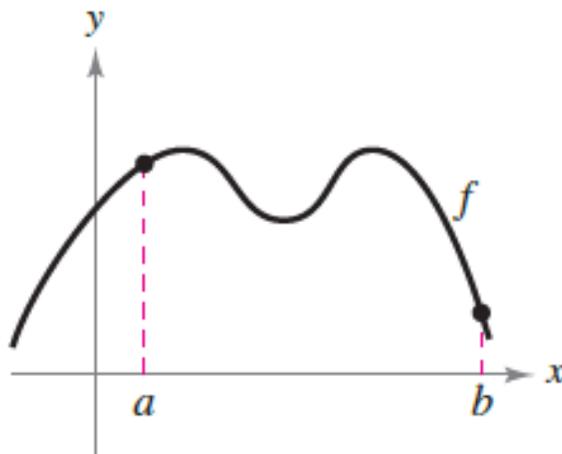
10. A ladder 13 meters long rests on horizontal ground and leans against a vertical wall. The top of the ladder is being pulled up the wall at 0.1 meters per second. How fast is the foot of the ladder approaching the wall when the foot of the ladder is 5 m from the wall? (5 pts)

11. Consider solving the equation $\sin x = x^2 - 3x + 1$ using Newton's Method.   (4 points total)

a) Write out the **particular** Newton's Method iteration, x_{n+1} for **this** problem. Hint: move all equations to one side and apply Newton's method on that function. (2 points)

x^2 -3x +1

sin x
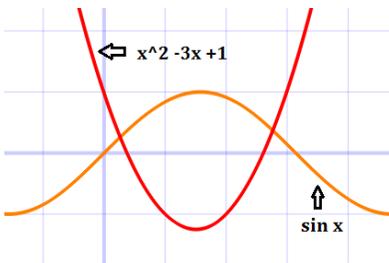
b) Find the two roots up with the given starting points. Write the decimal values only. (2 points)

$x_0 = 5$ $x_o = -2$